

Online-Appendix to the Article Strategic Effects of Regulatory Capital Requirements in Imperfect Banking Competition

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Appendix

Lemma 1. *If there exists a continuum of homogenous borrowers whose utility can be described with a representative utility function of the form $U(L_1, L_2) = m + a(L_1 + L_2) - \frac{b}{2}(L_1^2 + L_2^2) - dL_1L_2$, one can derive a generalized inverse demand function:*

$$r_i(L_i, L_j) = a - bL_i - dL_j. \quad (1)$$

Provided that both banks do not ration their customers and make non-negative profits, the direct demand function is defined as:

$$L_i(r_i, r_j) = \frac{a}{(b+d)} - \frac{br_i}{(b^2-d^2)} + \frac{dr_j}{(b^2-d^2)}. \quad (2)$$

Proof. We adopt the model of ? by assuming there is a continuum of borrowers of the same type with the following representative utility function, which is represented as:

$$U(L_1, L_2) = m + a(L_1 + L_2) - \frac{b}{2}(L_1^2 + L_2^2) - dL_1L_2$$

Where m represents all other goods and $p_m = 1$ is the normalized loan rate for all other goods. The representative borrower tries to maximize his utility subject to the budget constraint $Y \geq m + r_1L_1 + r_2L_2$ where Y is the income of the borrower. The Lagrangian function to describe the optimization problem is as follows:

$$\begin{aligned} \max \mathcal{L}(L_1, L_2) = \\ m + a(L_1 + L_2) - \frac{b}{2}(L_1^2 + L_2^2) - dL_1L_2 + \lambda(Y - m - r_1L_1 - r_2L_2) \end{aligned}$$

The Kuhn–Tucker conditions are:

$$a - bL_1 - dL_2 - \lambda r_1 \leq 0 \quad L_1 \geq 0 \quad L_1(a - bL_1 - dL_2 - \lambda r_1) = 0 \quad (3)$$

$$a - bL_2 - dL_1 - \lambda r_2 \leq 0 \quad L_2 \geq 0 \quad L_2(a - bL_2 - dL_1 - \lambda r_2) = 0 \quad (4)$$

$$1 - \lambda \leq 0 \quad m \geq 0 \quad m(1 - \lambda) = 0 \quad (5)$$

$$Y - m - r_1L_1 - r_2L_2 \leq 0 \quad \lambda \geq 0 \quad \lambda(Y - m - r_1L_1 - r_2L_2) = 0 \quad (6)$$

If the borrower's income Y is sufficiently large, such that $m > 0$, from 5 we obtain that:

$$\lambda = 1 \quad (7)$$

which implies the equation below with (6):

$$Y = m + r_1L_1 + r_2L_2 \quad (8)$$

Substituting (7) in (3) and (4), we obtain the following:

$$a - bL_1 - dL_2 - r_1 \leq 0 \quad L_1 \geq 0 \quad L_1(a - bL_1 - dL_2 - r_1) = 0 \quad (9)$$

$$a - bL_2 - dL_1 - r_2 \leq 0 \quad L_2 \geq 0 \quad L_2(a - bL_2 - dL_1 - r_2) = 0 \quad (10)$$

With positive demand for loans from both banks $L_1 > 0$ and $L_2 > 0$ we obtain the inverse demand curves below:

$$r_1(L_1, L_2) = a - bL_1 - dL_2 \quad (11)$$

$$r_2(L_1, L_2) = a - bL_2 - dL_1 \quad (12)$$

Since demand for loans of both banks is positive, we can invert the inverse demand curves in order to obtain the direct demand curves (note that $\frac{a(b-d)}{(b^2-d^2)} = \frac{a}{(b+d)}$), as follows:

$$L_1(r_1, r_2) = \frac{a}{(b+d)} - \frac{br_1}{(b^2-d^2)} + \frac{dr_2}{(b^2-d^2)} \quad (13)$$

$$L_2(r_1, r_2) = \frac{a}{(b+d)} - \frac{br_2}{(b^2-d^2)} + \frac{dr_1}{(b^2-d^2)} \quad (14)$$

If the demand for one bank is zero (i.e., $L_j = 0$) while the demand for the competitor is positive (i.e., $L_i > 0$), (9) and (10) imply that the inverse demand for loans from bank i is:

$$r_i(L_i) = a - bL_i$$

and the direct demand function that bank i faces is:

$$L_i(r_i) = \frac{a - r_i}{b}$$

At the same time, the Kuhn–Tucker conditions for the demand for loans j with the binding constraint $L_j = 0$ imply that:

$$r_j > a - dL_i.$$

□

Lemma 2. *A capital regulated bank that has raised capital in the first stage faces the following piecewise-defined cost function at the beginning of the second stage:*

$$C(L) = \begin{cases} r_e e & \text{if } L \leq e \\ r_D L + ce & \text{if } e < L \leq e/\delta \\ (r_D + c\delta)L + \theta(\delta L - e) & \text{if } L > e/\delta \end{cases} \quad (15)$$

Proof. Since the balance sheet constraint is binding, we can write D as a function of loans and capital $D = L - E - e$. If the bank invests in loans $L \leq e$, it can refund its assets solely out of the capital raised in the first stage at cost r_e . Hence, $D = 0$ and $E = 0$.

Intending to provide loans $L > e$, the bank needs to raise additional funds. Since refunding with deposits is cheaper (by assumption) than raising additional capital, the bank will refund the assets with deposits. No additional capital is required by regulation, in which $E = 0$ in the second stage. The cost of providing a loan amount $e < L < e/\delta$ is given by $r_e e + r_D(L - e)$. Since $r_e - r_D = c$, the cost of funding consists of the deposit rate times the total loan amount plus the risk premium for the capital raised in the first stage.

If the asset investment exceeds the first-stage loan capacity $L > e/\delta$, the bank is forced to raise additional capital in the second stage in order to comply with the minimum capital requirement. The additional capital is expressed as $E = \delta L - e$. Refunding cost can therefore be summarized as $r_D D + r_e e + r_E E$. Using the balance constraint leads to $r_D(L - \delta L) + (r_D + c)e + (r_D + c + \theta)\delta(L - e/\delta)$, which can be simplified to the third piece of the cost function. $\square$

Lemma 3. a) For given values of e_i, e_j, δ under the assumed demand system, a best response function $R_i(r_j)$ is represented as:

$$R_i(r_j) = \left\{ \begin{array}{ll} \frac{(b-d)a+dr_j}{2b} & =: R^I \\ \frac{(b-d)a-(b^2-d^2)e_i+dr_j}{b} & =: R^{II} \\ \frac{(b-d)a+dr_j+br_D}{2b} & =: R^{III} \\ \frac{(b-d)a-(b^2-d^2)\frac{e_i}{\delta}+dr_j}{b} & =: R^{IV} \\ \frac{(b-d)a+dr_j+b(r_D+\delta(c+\theta))}{2b} & =: R^V \\ \frac{br_j-(b-d)a}{d} & =: R^{VI} \\ \frac{1}{2}(a+r_d+\delta(c+\theta)) & =: R^{VII} \end{array} \right. ,$$

which is consistent if the following holds:

$$\begin{aligned}
R^I &\iff [0 \leq L_i(R_i(r_j), r_j) < e_i \quad \wedge \quad r_j < r_j^l(e_i)] \\
R^{II} &\iff [L_i(R_i(r_j), r_j) = e_i \quad \wedge \quad r_j^l(e_i) \leq r_j \leq r_j^h(e_i)] \\
R^{III} &\iff [e_i < L_i(R_i(r_j), r_j) < e_i/\delta \quad \wedge \quad r_j^h(e_i) < r_j < r_j^L(e_i)] \\
R^{IV} &\iff [L_i(R_i(r_j)_i, r_j) = e_i/\delta \quad \wedge \quad r_j^L(e_i) \leq r_j \leq r_j^H(e_i)] \\
R^V &\iff [e_i/\delta < L_i(R_i(r_j), r_j) \quad \wedge \quad r_j^H(e_i) < r_j \leq r_j^{L=0}(e_i)] \\
R^{VI} &\iff [L_j(R_i(r_j), r_j) = 0 \quad \wedge \quad r_j^{L=0} < r_j < r_j^{LM}] \\
R^{VII} &\iff [L_i(R_i(r_j)_i, r_j) \geq L_i^M \quad \wedge \quad r_j^{LM} \leq r_j].
\end{aligned}$$

b) Depending on the parameters of the model and the chosen capital in the first stage, a maximum of seven cases can occur, which are indicated above.

Proof. To prove part a) of Lemma 3, we proceed as follows. We first derive the first order condition for each piece of the profit function that implicitly defines the best loan rate response (i.e., R^I, R^{III}, R^V) for each of the cases discussed. Then we consider the points of discontinuity (R^{II}, R^{IV}). We then discuss the remaining possible cases (R^{VI}, R^{VII}). Finally, we show that part b) of Lemma 3 must hold.

From Lemma 2 we obtain bank i's piecewise-defined objective function $\max_{r_i} \Pi_i(r_i, r_j)$. A best response function $R_i(r_j)$ is implicitly defined by $\Pi'_i(R_i(r_j), r_j) = 0$. The first order conditions for the three pieces of bank i's objective function can be summarized as $(r_i - MC(L))L'_i + L_i = 0$ with $L'_i = -\frac{b}{b^2-d^2}$. Substituting L'_i and solving for r_i gives:

$$r_i = \frac{(b-d)a + dr_j + b(MC)}{2b} \quad (16)$$

Inserting the piecewise defined marginal cost into the best response function gives R^I, R^{III}, R^V :

R^I: If the best response to r_j results in a residual demand $0 \leq L(R_i(r_j), r_j) < e_i$, the marginal costs in the second stage are $MC(L(p)) = 0$. Substituting and solving for r_i gives $R_i(r_j) = \frac{(b-d)a + dr_j}{2b}$. We now have to show that $L_i(R_i(r_j), r_j) < e_i \forall r_j < r_j^l(e_i) := \frac{2(b^2-d^2)e_i - (b-d)a}{d}$. In closed form this can be rewritten as $e_i > \frac{a}{(b+d)} - \frac{b}{(b^2-d^2)} \frac{(b-d)a + dr_j}{2b} +$

$\frac{dr_j}{(b^2-d^2)}$. This can be simplified to $2e_i(b^2-d^2) > a(b-d) + dr_j$, or $\frac{2(b^2-d^2)e_i-(b-d)a}{d} > r_j$, which holds $\forall r_j < r_j^L(e_i)$.

R^{III}: If the best response to r_j results in a residual demand $e_i < L_i(R_i(r_j), r_j) < \frac{e_i}{\delta}$, the second-stage marginal cost are $MC(L(p)) = r_D$, which results in the best reaction $R_i(r_j) = \frac{(b-d)a+dr_j+br_D}{2b}$. We first show that $e_i < L_i(R_i(r_j), r_j)$ holds $\forall r_j^h(e_i) := \frac{2(b^2-d^2)e_i-(b-d)a+br_D}{d} < r_j < r_j^L(e_i) := \frac{2(b^2-d^2)e_i/\delta-(b-d)a}{d}$. The closed form is $e_i < \frac{a}{(b+d)} - \frac{b}{(b^2-d^2)} \frac{(b-d)a+dr_j+br_D}{2b} + \frac{dr_j}{(b^2-d^2)}$, which can be simplified to $2e_i(b^2-d^2) < a(b-d) - br_D + dr_j$, or $\frac{2(b^2-d^2)e_i-(b-d)a+br_D}{d} < r_j$, which holds $\forall r_j > r_j^h(e_i)$.

Similarly, it can be shown that $L_i(R_i(r_j), r_j) < e_i/\delta$, with $\frac{a}{(b+d)} - \frac{b}{(b^2-d^2)} \frac{(b-d)a+dr_j+br_D}{2b} + \frac{dr_j}{(b^2-d^2)} < e_i/\delta$ is true for $\forall r_j < r_j^L(e_i)$

R^V: If the best response to r_j results in a residual demand, represented as

$$L_i(R_i(r_j), r_j) > \frac{e_i}{\delta}, \text{ the optimal loan rate response is } R_i(r_j) = \frac{(b-d)a+dr_j+b(r_D+\delta(c+\theta))}{2b}.$$

This is the best loan rate response if $\frac{a}{(b+d)} - \frac{b}{(b^2-d^2)} \frac{(b-d)a+dr_j+b(r_D+\delta(c+\theta))}{2b} + \frac{dr_j}{(b^2-d^2)} > \frac{e_i}{\delta}$ or

$$\frac{2(b^2-d^2)e_i-(b-d)a+b(r_D+\delta(c+\theta))}{d} < r_j \text{ which is obviously true for all}$$

$$r_j > r_j^H(e_i) := \frac{2(b^2-d^2)e_i/\delta-(b-d)a+b(r_D+\delta(c+\theta))}{d} \text{ as long as the non-negativity constraints of the maximization program are not binding.}$$

R^{II} and **R^{IV}** result from the discontinuous jumps in the marginal cost function at the points where $L_i(R_i(r_j), r_j) = e_i$ and $L_i(R_i(r_j), r_j) = \frac{e_i}{\delta}$. These equalities define the capital and capacity clearing loan rate pairs (r_i, r_j) . Since $\forall r_j < r_j^L(e_i)$, the best loan rate response is $L_i(R_i(r_j), r_j) < e_i$ and $\forall r_j^h(e_i) < r_j < r_j^L(e_i)$ the best loan rate respond is $e_i < L_i(R_i(r_j), r_j)$ for $r_j^L(e_i) \leq r_j \leq r_j^h(e_i)$. Therefore, it must hold that $L_i(R_i(r_j), r_j) = e_i$ similar, for $r_j^L(e_i) \leq r_j \leq r_j^H(e_i)$ it must hold that $L_i(R_i(r_j), r_j) = \frac{e_i}{\delta}$.

R^{VI}: This branch of the best response function results from the assumptions of our model, i.e., the non-negativity constraints on demand in the optimization problem: $0 \leq L_j = \frac{a}{b+d} + \frac{br_j}{b^2-d^2} + \frac{dr_i}{b^2-d^2}$. A loan rate pair $(R_i(r_j), r_j)$ that result in a negative demand for

the opponent j makes the non-negativity constraint binding. Solving the constraint for r_i yields $R_i(r_j) = \frac{br_j - (b-d)a}{d}$. Inserting $L_j(R_i(r_j), r_j) = \frac{a}{(b+d)} - \frac{b}{(b^2-d^2)}r_j + \frac{d}{(b^2-d^2)}\frac{br_j - (b-d)a}{d}$ gives $L_j(R_i(r_j), r_j) = 0$.

R^{VII} : If $r_j > r_j^{L=0}$, bank i 's best response loan rate is only increasing in r_j as long as the “residual demand” (note that bank i covers the entire market since $L_j(R_i(r_j^{L=0}), r_j^{L=0}) = 0$) is smaller than the monopoly output $L_i(R_i(r_j), r_j) < L_i^M$. Hence, it is only optimal to respond to a further increase of bank r_j with increasing r_i as long as $R_i(r_j) > r_i^M$ with r_i^M being the monopoly loan rate that solves the maximization problem:

$r_i^M = \operatorname{argmax}(r_i - r_D - \delta(c + \theta))L_i r_i + \theta e_i$ implicitly defined by the first order condition. Solving for the loan rate gives $r_i^M = \frac{1}{2}(a + r_d + \delta(c + \theta))$.

Since r_i^M is unique and independent of r_j (depending solely on the exogenous model parameters), the best response loan rate function becomes vertical. It would never be beneficial to deviate from the monopoly loan rate for $L_i(R_i(r_j), r_j) < L_i^M$. Hence, no further best response branches can exist, which means that part b) must hold. $\square$

The critical values of the opponent's loan rate are implicitly defined by the intercept of the different best response branches. Equating the different branches and solving for r_j we derive the critical values:

$r_j^l(e_i) : (R^I = R^{II})$ solving $\frac{(b-d)a + dr_j}{2b} = \frac{(b-d)a - (b^2-d^2)e_i + dr_j}{b}$ for r_j yields

$$r_j^l(e_i) := \frac{2(b^2 - d^2)e_i - (b - d)a}{d}$$

$r_j^h(e_i) : (R^{II} = R^{III})$ solving $\frac{(b-d)a - (b^2-d^2)e_i + dr_j}{b} = \frac{(b-d)a + dr_j + br_D}{2b}$ for r_j yields

$$r_j^h(e_i) := \frac{2(b^2 - d^2)e_i - (b - d)a + br_D}{d}$$

$r_j^L(e_i) : (R^{III} = R^{IV})$ solving : $\frac{(b-d)a + dr_j + br_D}{2b} = \frac{(b-d)a - (b^2-d^2)\frac{e_i}{\delta} + dr_j}{b}$ for r_j yields

$$r_j^L(e_i) := \frac{2(b^2 - d^2)e_i/\delta - (b - d)a + br_D}{d}$$

$$r_j^H(e_i) : (R^{IV} = R^V) \text{ solving } \frac{(b-d)a - (b^2-d^2)e_i + dr_j}{b} = \frac{(b-d)a + dr_j + b(r_D + \delta(c+\theta))}{2b} \text{ for } r_j$$

yields

$$r_j^H(e_i) := \frac{2(b^2 - d^2)e_i/\delta - (b-d)a + b(r_D + \delta(c+\theta))}{d}$$

$$r_j^{L=0}(e_i) : (R^V = R^{VI}) \text{ solving } \frac{(b-d)a + dr_j + b(r_D + \delta(c+\theta))}{2b} = \frac{br_j - (b-d)a}{d} \text{ for } r_j \text{ yields}$$

$$r_j^{L=0}(e_i) := a - \frac{bd(a - (r_D + \delta(c+\theta)))}{2b^2 - d^2}$$

$$r_j^{L^M}(e_i) : (R^V = R^{VI}) \text{ solving } \frac{br_j - (b-d)a}{d} = \frac{1}{2}(a + r_d + \delta(c+\theta)) \text{ for } r_j \text{ yields}$$

$$r_j^{L^M}(e_i) := \frac{a(2b-d) + d(r_D + \delta(c+\theta))}{2b}$$